

# An Effective Field Theory Look at Deep Inelastic Scattering

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## Abstract

This talk discusses the effective field theory view of deep inelastic scattering. In such an approach, the standard factorization formula of a hard coefficient multiplied by a parton distribution function arises from matching of QCD onto an effective field theory. The DGLAP equations can then be viewed as the standard renormalization group equations that determines the cut-off dependence of the non-local operator whose forward matrix element is the parton distribution function. As an example, the non-singlet quark splitting functions is derived directly from the renormalization properties of the non-local operator itself. This approach, although discussed in the literature, does not appear to be well known to the larger high energy community. In this talk we give a pedagogical introduction to this subject.

# 1 Introduction

Electron-nucleon Deep Inelastic Scattering (DIS) has played an important role in our understanding of the strong interactions. By now, it is a standard topic in particle physics and quantum field theory books [1, 2].

The cross section for this process can be written in term of two structure functions. These obey a factorization formula (see for example [1, 3])

$$F_2(x, Q^2) = \sum_i \int d\xi C_2^i \left( \frac{x}{\xi}, \frac{Q}{\mu}, \frac{\mu_F}{\mu}, \alpha_s(\mu) \right) \phi_{i/h}(\xi, \mu_F, \alpha_s(\mu)) + \mathcal{O} \left( \frac{\Lambda_{\text{QCD}}^2}{Q^2} \right). \quad (1)$$

The parton distribution function (PDF)  $\phi_{i/h}$  is often referred to as the “probability to find a parton  $i$  in hadron  $h$ ”. Such a description, although very intuitive, requires a field theoretic definition. What do we mean by a “distribution function”?

This question was answered in the early 1980’s [4, 5]. The quark PDF is

$$\phi_{i/h}(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{ixn \cdot pt} \langle h(p) | \bar{\psi}_i(0) W_n(0) W_n^\dagger(tn) \frac{\not{n}}{2} \psi_i(tn) | h(p) \rangle \Big|_{\text{avg.}}, \quad (2)$$

where  $n$  light-like vector,  $n^2 = 0$  and  $W_n$  is a light-like Wilson line,

$$W_n(u) = P \exp ig \int_{-\infty}^0 ds n \cdot A(u + sn), \quad (3)$$

and  $A_\mu \equiv A_\mu^a t^a$ , with  $t^a$  in the fundamental representation.

Let us discuss (2) in more detail. First, in the definition of the PDF, an average over color and spin is understood. Second, since  $\bar{\psi}_i$  and  $\psi_i$  are not at the same point, we must include a “gauge-link” to render the operator gauge invariant. This is the reason that the Wilson lines appear in the definition of the PDF. Notice that the product of the two semi-infinite Wilson lines can be written as finite Wilson line connecting the point 0 and  $tn$ . Third, the PDF is just the Fourier transform of the diagonal matrix element of a standard dimension-three operator. Since the operator is non-local, its Fourier transform is a function and not a number. Fourth, the anti-quark PDF and gluon PDF can be defined in a similar way to the quark PDF. For example, the gluon PDF is defined with the quark field replaced by gluon field strength and the Wilson line is in the adjoint representation. For concreteness in the following we will focus on the quark PDF.

The PDF satisfies the DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) equation

$$\frac{d\phi_{i/h}(x)}{d \ln \mu} = \int_x^1 \frac{d\xi}{\xi} P_{ij}(\xi, \alpha_s) \phi_{j/h}(x), \quad (4)$$

where  $P_{ij}$  are the so called “splitting functions”. These are usually calculated via two main methods.

The first is based on the interpretation of the splitting function as the “probability” of the quark to “split” into a quark and a gluon [6]. At  $\mathcal{O}(\alpha_s)$  one finds

$$“P_{qq}” = \frac{C_F \alpha_s}{\pi} \frac{1 + \xi^2}{1 - \xi},$$

where  $C_F = 4/3$  for QCD. The full expression is in fact

$$P_{qq} = \frac{C_F \alpha_s}{\pi} \left[ (1 + \xi^2) \frac{1}{(1 - \xi)_+} + \frac{3}{2} \delta(1 - \xi) \right] + \mathcal{O}(\alpha_s^2) \equiv \frac{C_F \alpha_s}{\pi} P_{qq}^{[0]} + \mathcal{O}(\alpha_s^2). \quad (5)$$

and the singular terms  $\frac{1}{(1 - \xi)_+}$ , defined in the appendix, and  $\delta(1 - \xi)$  must be added “by hand”.

The second method, often called the “OPE” method [7, 8], is based on the fact that moments of the PDF are matrix elements of local operators. Thus one defines the moments

$$\int_0^1 dx x^{n-1} \phi(x) = \phi_n$$

which satisfy a local renormalization group equation (RGE)

$$\frac{d\phi_n}{d \ln \mu} = -\gamma_n \phi_n.$$

The splitting function is related to  $\gamma_n$  via

$$\gamma_n = - \int_0^1 dx x^{n-1} P_{qq}.$$

There is also another approach possible, which is less known, and can be described as the “effective field theory” method. Going back to the factorization formula for  $F_2$ , we can write it schematically as

$$F_2 = \underset{\substack{\uparrow \\ \text{Wilson coef.}}}{C_2(\mu)} \otimes \underset{\substack{\uparrow \\ \text{Operator}}}{\phi(\mu)} + \text{power corrections} \quad (6)$$

The factorization formula can be interpreted as a result of matching QCD onto an effective field theory.  $C_2$  is the Wilson coefficient extracted in the process, and the PDF is the matrix element of the operator in the effective theory. The “power corrections” correspond to the contribution of power suppressed operators. This is the standard expression one finds in a generic effective field theory approach. The only complication in this case is the non-locality of the operator. The scale  $\mu$  on which the PDF depends is then just the cut-off scale of the effective theory. It is not surprising then that the cut-off dependence of the PDF is determined by an RGE which is schematically

$$\frac{d\phi}{d \ln \mu} = P \otimes \phi.$$

This is none other than the famous DGLAP equation, where the splitting function is simply the anomalous dimension. Since the PDF is a function, the anomalous dimension is a function too, and not a constant.

If the splitting function is just the anomalous dimension of the PDF, one can calculate it as one usually does in field theory. In particular, if we use dimensional regularization and a mass independent scheme, the anomalous dimension can be extracted from the  $1/\epsilon$  pole of the loop corrections to the PDF. The only unusual aspect of the calculation arises from the non-local structure of the PDF. We will discuss this calculation in detail in the next section.

This calculation was first performed, to the best of our knowledge, by Braunschweig, Horejsi and Robaschik [9]. But their paper is almost unknown. By the end of 2009 it had 5 citations on SPIRES [10]. For comparison, by the end of 2009 reference [6] had 3933 citations while references [7] and [8] had 589 and 919 citations, respectively. The situation is slightly better than these citation counts indicate, but still the third approach is much less known, especially to the general high energy community.

The effective field theory approach has received much interest in the recent years, since the effective field theory one is matching onto is the soft collinear effective theory (SCET) [11, 12, 13]. Calculations of the anomalous dimension of non-local operators are standard in SCET. The application of SCET to DIS, as well as other hard QCD processes, was first discussed in [14]. In that paper the matching coefficients were calculated at tree level. Although SCET is the appropriate EFT for DIS, the effect of the soft gluons completely cancel for generic  $x$  [14, 15]. In this case SCET is just “boosted QCD” and we can calculate the anomalous dimension by using QCD Feynman rules instead of SCET Feynman rules. We will demonstrate this approach by calculating  $P_{qq}$  for the non-singlet case. The method will be technically different from [9], but conceptually very similar. For another pedagogical discussion of the calculation of the evolution (RGE) equations for non-local operators see appendix G of [16], where the calculation techniques are demonstrated for both covariant and light-cone gauges.

## 2 Example of a Non-Local Renormalization: Non-singlet Splitting Function

### 2.1 Feynman rules

We begin by deriving the Feynman rules. We will use the definition of the PDF in equation (2), and replace the hadron state by a free quark. Since we are interested in the cut-off (UV) dependence of the PDF which is independent of the IR physics, such a replacement is justified. The “zero-gluon” Feynman rule is

$$\begin{aligned}
\text{Diagram: } \begin{array}{c} \nearrow \\ \nwarrow \end{array} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i\xi n \cdot pt} \langle k | \bar{\psi}(0) \frac{\not{n}}{2} \psi(tn) | k \rangle \\
&= \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i\xi n \cdot pt} e^{i0 \cdot k} e^{-itn \cdot k} \bar{u}(k) \frac{\not{n}}{2} u(k) \\
&= \delta(\xi n \cdot p - n \cdot k) \bar{u}(k) \frac{\not{n}}{2} u(k).
\end{aligned}$$

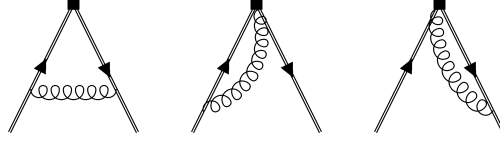
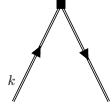


Figure 1: Feynman diagrams

Taking the external states to carry momentum  $p$  we find that the zeroth order expression is

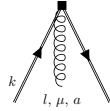
$$D_0 = \frac{1}{n \cdot p} \delta(1 - \xi) \bar{u}(p) \frac{\not{p}}{2} u(p). \quad (7)$$

Averaging over the spins we would find  $\delta(\xi - 1)$ , i.e. the quark is carrying all of the momentum of the hadron. The zero-gluon Feynman rule is therefore



$$\delta(\xi n \cdot p - n \cdot k) \frac{\not{p}}{2}.$$

In order to find the one-gluon Feynman rule we need to expand the Wilson lines in (3). A simple calculation yields



$$t^a g \frac{n^\mu}{n \cdot l} [\delta(\xi n \cdot p - n \cdot k) - \delta(\xi n \cdot p - n \cdot k + n \cdot l)] \frac{\not{p}}{2}$$

## 2.2 Feynman integration

In order to find  $P_{qq}$  we need to calculate three diagrams, see Figure 1. To demonstrate the techniques, we will calculate the leftmost diagram in Figure 1. We will use Feynman gauge in the calculation.

First we need to define light-cone coordinates. We define two light-cone vectors  $n^2 = \bar{n}^2 = 0$ ,  $n \cdot \bar{n} = 2$ . In particular we will use  $n = (1, 0, 0, -1)$ ,  $\bar{n} = (1, 0, 0, 1)$ . Notice that  $n \cdot p = p_0 + p_3 \geq 0$ . Any four vector can be decomposed as

$$a^\mu = \bar{n} \cdot a \frac{n^\mu}{2} + n \cdot a \frac{\bar{n}^\mu}{2} + a_\perp^\mu, \quad (8)$$

which also defines the “ $\perp$ ” coordinates. In these coordinates  $d^d k$  can be written as

$$d^d k = \frac{1}{2} dn \cdot k d\bar{n} \cdot k d^{d-2} k_\perp.$$

We can now write down the Feynman integral,

$$I = \int \frac{1}{2(2\pi)^d} dn \cdot k d\bar{n} \cdot k d^{d-2} k_\perp \left[ \frac{1}{k^2 + i0} \right]^2 \frac{\text{Num.}}{(k - p)^2 + i0} \delta(\xi n \cdot p - n \cdot k), \quad (9)$$

where  $p$  ( $k$ ) is the external (internal) quark momentum and

$$\text{Num.} = (n \cdot k)^2 \frac{\not{n}}{2} + n \cdot k \not{k}_\perp - k_\perp^2 \frac{\not{n}}{2}.$$

The total diagram is given by

$$D_1 = -ig^2 C_F \mu^{2\epsilon} \bar{u}(p) \gamma^\mu I \gamma_\mu u(p) \quad (10)$$

The only difference from the usual Feynman integral is the presence of the delta function. It also forces us to split the integration according to the light-cone coordinates. Our strategy will be to calculate the  $\bar{n} \cdot k$  integral using residue theorem, to calculate the  $k_\perp$  integral in  $d-2$  dimensions, and use the delta function to perform the integral over  $n \cdot k$  [17].

In order to use the residue theorem for the  $\bar{n} \cdot k$  integral we need to find the poles of the integrand. They are given by

$$\begin{aligned} k^2 + i0 = \bar{n} \cdot k n \cdot k + k_\perp^2 + i0 = 0 &\Rightarrow \bar{n} \cdot k = \frac{-k_\perp^2 - i0}{n \cdot k} \\ (k-p)^2 + i0 = 0 &\Rightarrow \bar{n} \cdot k = \bar{n} \cdot p + \frac{-(k-p)_\perp^2 - i0}{n \cdot k - n \cdot p}. \end{aligned} \quad (11)$$

In order to get a non-zero result we must ensure that the two poles are on the opposite sides of the real  $\bar{n} \cdot k$  axis. Otherwise we can close the contour in the half plane that contains no poles and get zero. There are two options then,

$$\begin{aligned} \text{I)} \quad n \cdot k > 0 \quad \cup \quad n \cdot k - n \cdot p < 0 &\Rightarrow 0 < n \cdot k < n \cdot p \Rightarrow 0 < \frac{n \cdot k}{n \cdot p} < 1 \\ \text{II)} \quad n \cdot k < 0 \quad \cup \quad n \cdot k - n \cdot p > 0 &\Rightarrow n \cdot p < n \cdot k < 0. \end{aligned} \quad (12)$$

Recall, though, that  $n \cdot p > 0$ , so the second option is ruled out. Combining the first option with the delta function  $\delta(\xi n \cdot p - n \cdot k)$  we find that  $0 < \xi < 1$ . Notice also that we have recovered the usual support property of  $\phi_{i/h}(\xi)$ , namely, that the momentum fraction carried by the quark must be between 0 and 1.

After performing the  $n \cdot k$  integral using residues, we perform the integral over  $k_\perp$  using dimensional regularization in  $d-2$  dimensions. Due to the numerator structure, we can split  $I$  into 3 integrals that differ by their Dirac structure. In order to find the anomalous dimension we only need the Dirac structure that appear in the definition of  $\phi_{i/h}(\xi)$ , i.e  $\not{n}$ . The other Dirac structures lead to finite terms and do not contribute to the anomalous dimension.<sup>1</sup>

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<sup>1</sup>We can verify this point using another argument. The integral can be organized by the so called method of regions [18, 19], which is also related to our effective field theory interpretation. The method of regions allows us to divide the momentum integration to various regions in which the integral momentum scales in a particular way. The sum of all the regions is equal to the full integral.

For our integrals there is only a collinear region, namely a region in which the components of the momentum  $k$  scale as

$$n \cdot k \sim \mathcal{O}(1), \quad \bar{n} \cdot k \sim \mathcal{O}(\Lambda_{\text{QCD}}^2), \quad k_\perp \sim \mathcal{O}(\Lambda_{\text{QCD}}).$$

We will therefore ignore the other Dirac structures in the numerator. In order to regulate the IR divergences we keep  $p^2 \neq 0$ . Performing the  $\bar{n} \cdot k$  integral for this structure by using the first pole in (11), we find

$$\begin{aligned} I_{\not{n}} &= -\frac{1}{2} \int \frac{1}{2(2\pi)^d} dn \cdot k d\bar{n} \cdot k d^{d-2}k_{\perp} \left[ \frac{1}{k^2 + i0} \right]^2 \frac{k_{\perp}^2 \not{n}}{(k-p)^2 + i0} \delta(\xi n \cdot p - n \cdot k) \\ &= i \frac{\not{n}}{2} \frac{1}{4\pi} \int_0^1 dr \delta(\xi - r) \int \frac{d^{d-2}k_{\perp}}{(2\pi)^{d-2}} \frac{1}{n \cdot p} \frac{(1-r)(k_{\perp} + rp_{\perp})^2}{[k_{\perp}^2 - (-p^2 - i0)r(1-r)]^2} \end{aligned} \quad (13)$$

where we have defined  $r = n \cdot k / n \cdot p$ . The limits of integration are determined by the  $\bar{n} \cdot k$  integral as explained above. Performing the relevant  $k_{\perp}$  integration, we find that the divergent part of the diagram is

$$\begin{aligned} D_1^{\text{div}} &= -\frac{1}{n \cdot p} \frac{C_F \alpha_s}{4\pi} \bar{u}(p) \gamma^{\mu} \frac{\not{n}}{2} \gamma_{\mu} u(p) (4\pi\mu^2)^{\epsilon} (1-\epsilon) \Gamma(\epsilon) (-p^2 - i0)^{-\epsilon} \\ &\quad \times \int_0^1 dr (1-r) [r(1-r)]^{-\epsilon} \delta(\xi - r), \end{aligned} \quad (14)$$

Expanding in  $\epsilon$  we finally find

$$D_1^{\text{div}} = \frac{1}{\epsilon} \frac{1}{n \cdot p} \frac{C_F \alpha_s}{4\pi} \bar{u}(p) \frac{\not{n}}{2} u(p) 2(1-\xi) \theta(\xi) \theta(1-\xi). \quad (15)$$

The other two diagrams can be calculated using the same methods. Using the one-gluon Feynman rule we find that

$$\begin{aligned} D_2 = D_3 &= \frac{1}{n \cdot p} \frac{C_F \alpha_s}{4\pi} \bar{u}(p) \frac{\not{n}}{2} u(p) (4\pi\mu^2)^{\epsilon} \Gamma(\epsilon) (-p^2 - i0)^{-\epsilon} \\ &\quad \times \left[ \xi^{1-\epsilon} (1-\xi)^{-1-\epsilon} \theta(1-\xi) \theta(\xi) - \delta(1-\xi) B(2-\epsilon, -\epsilon) \right]. \end{aligned} \quad (16)$$

In order to expand in  $\epsilon$  we need to use the identity [20], which is also proven in the appendix,

$$\theta(\xi) \theta(1-\xi) (1-\xi)^{-1-\epsilon} = -\frac{1}{\epsilon} \delta(1-\xi) + \left( \frac{1}{1-\xi} \right)_+ + \mathcal{O}(\epsilon). \quad (17)$$

Expanding in  $\epsilon$  we finally find that the divergent part is

$$D_2^{\text{div}} = D_3^{\text{div}} = \frac{1}{\epsilon} \frac{1}{n \cdot p} \frac{C_F \alpha_s}{4\pi} \bar{u}(p) \frac{\not{n}}{2} u(p) 2 \theta(\xi) \left[ \xi \frac{1}{(1-\xi)_+} + \delta(1-\xi) \right]. \quad (18)$$

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We also have

$$d^4k \sim \mathcal{O}(\Lambda_{\text{QCD}}^4), \quad k^2 \sim \mathcal{O}(\Lambda_{\text{QCD}}^2) \quad \Rightarrow I \sim \Lambda_{\text{QCD}}^4 \left( \frac{1}{\Lambda_{\text{QCD}}^2} \right)^3 \times \text{Num.}$$

Since the anomalous dimension can depend only on  $n \cdot p$ , it must be an  $\mathcal{O}(1)$  quantity, and so does  $I$ . We therefore need the numerator to scale as  $\Lambda_{\text{QCD}}^2$ , so only the  $k_{\perp}^2$  integral will contribute to the anomalous dimension. This integral involves only the  $\not{n}$  Dirac structure.

Notice that the singular terms  $\frac{1}{(1-\xi)_+}$  and  $\delta(1-\xi)$  arise naturally in the course of the calculation and are not added “by hand”.

We also need the wave function renormalization constant  $Z_Q$  [21] in Feynman gauge

$$Z_Q = 1 - \frac{C_F \alpha_s}{4\pi} \frac{1}{\epsilon}. \quad (19)$$

The total expression for the matrix element at one loop accuracy is

$$\begin{aligned} D_{\text{total}}^{\text{div}} &= D_1^{\text{div}} + D_2^{\text{div}} + D_3^{\text{div}} + Z_Q D_0 \\ &= \frac{1}{n \cdot p} \bar{u}(p) \frac{\not{p}}{2} u(p) \left\{ \delta(1-\xi) + \theta(\xi) \theta(1-\xi) \frac{C_F \alpha_s}{4\pi} \frac{2}{\epsilon} \left[ (1-\xi) + 2\xi \frac{1}{(1-\xi)_+} + \frac{3}{2} \delta(1-\xi) \right] \right\} \\ &= \frac{1}{n \cdot p} \bar{u}(p) \frac{\not{p}}{2} u(p) \left\{ \delta(1-\xi) + \theta(\xi) \theta(1-\xi) \frac{C_F \alpha_s}{4\pi} \frac{2}{\epsilon} \left[ (1+\xi^2) \frac{1}{(1-\xi)_+} + \frac{3}{2} \delta(1-\xi) \right] \right\}. \end{aligned} \quad (20)$$

Averaging over the spins we find that

$$\phi_{\text{bare}}(\xi) = \delta(1-\xi) + \frac{C_F \alpha_s}{4\pi} \frac{2}{\epsilon} \theta(\xi) \theta(1-\xi) \left[ (1+\xi^2) \frac{1}{(1-\xi)_+} + \frac{3}{2} \delta(1-\xi) \right] + \text{finite} + \mathcal{O}(\alpha_s^2) \quad (21)$$

## 2.3 Renormalization

We now calculate the RGE equation in the  $\overline{\text{MS}}$  scheme in the “standard way”, see for example [22]. We define the renormalization factor  $Z(x, \xi)$  by

$$\phi^{\text{bare}}(x) = \int_0^1 d\xi Z(x, \xi) \phi^{\text{ren.}}(\xi) \equiv Z(x, \xi) \otimes \phi^{\text{ren.}}(x), \quad (22)$$

and expand  $Z$  in  $\alpha_s/4\pi$ , such that  $Z_{[n]}$  denotes the coefficient of the  $(\alpha_s/4\pi)^n$  term. At tree level we have  $\phi_{[0]}^{\text{bare}}(x) = \phi_{[0]}^{\text{ren.}}(x) = \delta(1-x)$ , so

$$Z_{[0]}(x, \xi) = \delta(\xi - x) = \frac{1}{\xi} \delta\left(1 - \frac{x}{\xi}\right). \quad (23)$$

In order to calculate the one loop expression for the renormalization factor we use

$$\phi_{[1]}^{\text{bare}}(x) = Z_{[0]}(x, \xi) \otimes \phi_{[1]}^{\text{ren.}}(\xi) + Z_{[1]}(x, \xi) \otimes \phi_{[0]}^{\text{ren.}}(\xi)$$

to find

$$Z_{[1]}(x, \xi) = \theta\left(1 - \frac{x}{\xi}\right) \theta\left(\frac{x}{\xi}\right) \frac{1}{\epsilon} \frac{2}{\xi} P_{qq}^{[0]} \left( \frac{x}{\xi} \right), \quad (24)$$

or

$$Z(x, \xi) = \frac{1}{\xi} \delta\left(1 - \frac{x}{\xi}\right) + \frac{C_F \alpha_s}{4\pi} \theta\left(1 - \frac{x}{\xi}\right) \theta\left(\frac{x}{\xi}\right) \frac{1}{\epsilon} \frac{2}{\xi} P_{qq}^{[0]} \left( \frac{x}{\xi} \right) + \mathcal{O}(\alpha_s^2), \quad (25)$$



where  $P_{qq}^{[0]}$  is defined in (5). In order to calculate the anomalous dimension we now follow the standard procedure [22], treating  $Z$  and  $\gamma$  as infinite dimensional matrices. Define

$$O^{\text{bare}} = Z \otimes O^{\text{ren.}} \quad \text{and} \quad \frac{dO^{\text{ren.}}}{d \ln \mu} = \gamma \otimes O^{\text{ren.}}, \quad (26)$$

and expand  $Z$  as

$$Z = \mathbb{1} + \sum_{k=1}^{\infty} \frac{1}{\epsilon^k} Z^{(k)}. \quad (27)$$

The anomalous dimension  $\gamma$  is given by

$$\gamma = 2\alpha_s \frac{\partial Z^{(1)}}{\partial \alpha_s}. \quad (28)$$

In this way we find that

$$\gamma(x, \xi) = \frac{C_F \alpha_s}{\pi} \theta(1 - \frac{x}{\xi}) \theta(\frac{x}{\xi}) \frac{1}{\xi} P_{qq}^{[0]} \left( \frac{x}{\xi} \right) + \mathcal{O}(\alpha_s^2), \quad (29)$$

and the RGE is finally

$$\frac{d\phi(x)}{d \ln \mu} = \int_0^1 d\xi \gamma(x, \xi) \phi(\xi) = \int_x^1 \frac{d\xi}{\xi} P_{qq}^{[0]}(\xi) \phi\left(\frac{x}{\xi}\right) + \mathcal{O}(\alpha_s^2), \quad (30)$$

where we have changed  $\xi \rightarrow x/\xi$ . We have recovered the standard DGLAP equation at  $\mathcal{O}(\alpha_s)$ .

### 3 Conclusions

We have argued that the standard features of DIS, namely, factorization into a short distance coefficient times a parton distribution function and DGLAP evolution of the PDF, can be understood in the language of an effective field theory. In this interpretation QCD is matched onto an effective field theory. The short distance coefficient is simply the Wilson coefficient and the PDF is the matrix element of the operator in the effective field theory. The only difference from more familiar effective field theories is that the operator is non-local and therefore its matrix element is a function and not a number. The cut-off dependence of the matrix element can be determined in the standard way. In particular, by using dimensional regularization and a mass independent scheme, the anomalous dimension can be extracted from the  $1/\epsilon$  pole of the loop corrections to the PDF. The resulting RGE equations are none other than the famous DGLAP equations. This approach, although discussed in the literature, does not appear to be well known to the larger high energy community.

We have demonstrated this approach explicitly by calculating the one-loop correction to the non-singlet quark PDF. We have then derived the evolution equation from the divergent piece of the one-loop correction. This procedure can be generalized in several ways. First, we can calculate higher order corrections in  $\alpha_s$ . Second, we can include operator mixing, such as the calculation of  $P_{qg}$  and  $P_{gq}$ . Third, we can include power corrections. For example, one can

calculate in this method the anomalous dimension of twist 4 operators, which is still an open problem.

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## A The plus distribution

The “plus” distribution  $\frac{1}{(1-x)_+}$  is defined such that for any test function  $f(x)$

$$\int_0^1 dx f(x) \frac{1}{(1-x)_+} = \int_0^1 dx \frac{f(x) - f(1)}{(1-x)}. \quad (31)$$

More generally  $[h(x)]_+$  is defined as

$$\int_0^1 dx f(x) [h(x)]_+ = \int_0^1 dx (f(x) - f(1)) h(x). \quad (32)$$

When multiplying  $[h(x)]_+$  by a function  $g(x)$  which is regular at  $x = 1$  we have

$$\int_0^1 dx f(x) g(x) [h(x)]_+ = \int_0^1 dx (f(x)g(x) - f(1)g(1)) h(x). \quad (33)$$

Notice that

$$[g(x)h(x)]_+ = g(x)[h(x)]_+ - \delta(1-x) \int_0^1 dx (g(x) - g(1)) h(x). \quad (34)$$

Another useful property is the identity

$$(1-x)^n \frac{1}{(1-x)_+} = (1-x)^{n-1}. \quad (35)$$

Thus we can write

$$P_{qq}^{[0]} = (1-\xi) + 2\xi \frac{1}{(1-\xi)_+} + \frac{3}{2} \delta(1-\xi) = (1+\xi^2) \frac{1}{(1-\xi)_+} + \frac{3}{2} \delta(1-\xi) = \left( \frac{1+\xi^2}{1-\xi} \right)_+. \quad (36)$$

In order to prove the identity

$$\theta(x)\theta(1-x)(1-x)^{-1-\epsilon} = -\frac{1}{\epsilon} \delta(1-x) + \left( \frac{1}{1-x} \right)_+ + \mathcal{O}(\epsilon), \quad (37)$$

we multiply  $(1-x)^{-1-\epsilon}$  by a test function,  $f(x)$  and integrate

$$\int_0^1 dx f(x) (1-x)^{-1-\epsilon} = \int_0^1 dx [f(x) - f(1)] (1-x)^{-1-\epsilon} + f(1) \int_0^1 dx (1-x)^{-1-\epsilon} \quad (38)$$

Both terms on the right hand side are regular at  $x = 1$ , so we can expand in  $\epsilon$  and find

$$\int_0^1 dx f(x) (1-x)^{-1-\epsilon} = \int_0^1 dx \frac{f(x) - f(1)}{1-x} - \frac{f(1)}{\epsilon} + \mathcal{O}(\epsilon). \quad (39)$$

Notice that

$$\theta(x)\theta(1-x)g(x)(1-x)^{-1-\epsilon} = -\frac{1}{\epsilon}\delta(1-x)g(1) + g(x)\left(\frac{1}{1-x}\right)_+ + \mathcal{O}(\epsilon). \quad (40)$$

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